

# Interacting holographic phantom

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**Abstract.** In this paper we consider the holographic model of interacting dark energy in non-flat universe. With the choice of  $c \leq 0.84$ , the interacting holographic dark energy can be described by a phantom scalar field. Then we show this phantom description of the holographic dark energy with  $c \leq 0.84$  and reconstruct the potential of the phantom scalar field.

## 1 Introduction

Nowadays it is strongly believed that the universe is experiencing an accelerated expansion. Recent observations from type Ia supernovae [1–3] in associated with large scale structure [4, 5] and cosmic microwave background anisotropies [6, 7] have provided main evidence for this cosmic acceleration. In order to explain why the cosmic acceleration happens, many theories have been proposed. Although theories of trying to modify Einstein equations constitute a big part of these attempts, the mainstream explanation for this problem, however, is known as theories of dark energy. It is the most accepted idea that a mysterious dominant component, dark energy, with negative pressure, leads to this cosmic acceleration, though its nature and cosmological origin still remain enigmatic at present.

The combined analysis of cosmological observations suggests that the universe consists of about 70% dark energy, 30% dust matter (cold dark matter plus baryons), and negligible radiation. Although the nature and origin of dark energy are unknown, we still can propose some candidates to describe it. The most obvious theoretical candidate of dark energy is the cosmological constant  $\lambda$  (or vacuum energy) [8–13] which has the equation of state  $w = -1$ . However, as is well known, there are two difficulties arise from the cosmological constant scenario, namely the two famous cosmological constant problems – the “fine-tuning” problem and the “cosmic coincidence” problem [14].

An alternative proposal for dark energy is the dynamical dark energy scenario. The cosmological constant puzzles may be better interpreted by assuming that the vacuum energy is canceled to exactly zero by some unknown mechanism and introducing a dark energy component with a dynamically variable equation of state. The dynamical dark energy proposal is often realized by some scalar field mechanism which suggests that the energy

form with negative pressure is provided by a scalar field evolving down a proper potential. So far, a large class of scalar-field dark energy models have been studied, including quintessence [15–24],  $K$ -essence [25, 26], tachyon [27, 28], phantom [29–32], ghost condensate [33, 34] and quintom [35–41], and so forth. But we should note that the mainstream viewpoint regards the scalar field dark energy models as an effective description of an underlying theory of dark energy. In addition, other proposals on dark energy include interacting dark energy models [42–46], braneworld models [47, 48], and Chaplygin gas models [49], etc.. One should realize, nevertheless, that almost these models are settled at the phenomenological level, lacking theoretical root.

It is generally believed by theorists that we can not entirely understand the nature of dark energy before a complete theory of quantum gravity is established [50]. However, although we are lacking a quantum gravity theory today, we still can make some attempts to probe the nature of dark energy according to some principles of quantum gravity. The holographic dark energy model is just an appropriate example, which is constructed in the light of the holographic principle of quantum gravity theory. That is to say, the holographic dark energy model possesses some significant features of an underlying theory of dark energy.

The distinctive feature of the cosmological constant or vacuum energy is that its equation of state is always exactly equal to  $-1$ . However, when considering the requirement of the holographic principle originating from the quantum gravity speculation, the vacuum energy will acquire dynamically property. As we speculate, the dark energy problem may be in essence a problem belongs to quantum gravity [50]. In the classical gravity theory, one can always introduce a cosmological constant to make the dark energy density be an arbitrary value. However, a complete theory of quantum gravity should be capable of making the properties of dark energy, such as the energy density and the equation of state, be determined definitely and uniquely. Currently, an interesting attempt for probing the

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nature of dark energy within the framework of quantum gravity is the so-called “holographic dark energy” proposal [51–55]. It is well known that the holographic principle is an important result of the recent researches for exploring the quantum gravity (or string theory) [56, 57]. This principle is enlightened by investigations of the quantum property of black holes. Roughly speaking, in a quantum gravity system, the conventional local quantum field theory will break down. The reason is rather simple: For a quantum gravity system, the conventional local quantum field theory contains too many degrees of freedom, and such many degrees of freedom will lead to the formation of black hole so as to break the effectiveness of the quantum field theory.

For an effective field theory in a box of size  $L$ , with UV cut-off  $\Lambda$  the entropy  $S$  scales extensively,  $S \sim L^3 \Lambda^3$ . However, the peculiar thermodynamics of black hole [58–60] has led Bekenstein to postulate that the maximum entropy in a box of volume  $L^3$  behaves nonextensively, growing only as the area of the box, i.e. there is a so-called Bekenstein entropy bound,  $S \leq S_{\text{BH}} \equiv \pi M_{\text{P}}^2 L^2$ . This nonextensive scaling suggests that quantum field theory breaks down in large volume. To reconcile this breakdown with the success of local quantum field theory in describing observed particle phenomenology, Cohen et al. [51] proposed a more restrictive bound – the energy bound. They pointed out that in quantum field theory a short distance (UV) cut-off is related to a long distance (IR) cut-off due to the limit set by forming a black hole. In other words, if the quantum zero-point energy density  $\rho_\Lambda$  is relevant to a UV cut-off  $\Lambda$ , the total energy of the whole system with size  $L$  should not exceed the mass of a black hole of the same size, thus we have  $L^3 \rho_\Lambda \leq L M_{\text{P}}^2$ . This means that the maximum entropy is in order of  $S_{\text{BH}}^{3/4}$ . When we take the whole universe into account, the vacuum energy related to this holographic principle [56, 57] is viewed as dark energy, usually dubbed holographic dark energy. The largest IR cut-off  $L$  is chosen by saturating the inequality so that we get the holographic dark energy density

$$\rho_\Lambda = 3c^2 M_{\text{P}}^2 L^{-2}, \quad (1)$$

where  $c$  is a numerical constant, and  $M_{\text{P}} \equiv 1/\sqrt{8\pi G}$  is the reduced Planck mass. Many authors have devoted to developed the idea of the holographic dark energy. It has been demonstrated that it seems most likely that the IR cutoff is relevant to the future event horizon

$$R_{\text{h}}(a) = a \int_t^\infty \frac{dt'}{a(t')} = a \int_a^\infty \frac{da'}{H a'^2}. \quad (2)$$

Such a holographic dark energy looks reasonable, since it may provide simultaneously natural solutions to both dark energy problems as demonstrated in [55]. The holographic dark energy model has been tested and constrained by various astronomical observations [61–64]. Furthermore, the holographic dark energy model has been extended to include the spatial curvature contribution, i.e. the holographic dark energy model in non-flat space [65]. We focus in this paper on the holographic dark energy in a non-flat universe. For other extensive studies, see e.g. [66–76].

It is known that the coincidence or “why now” problem is easily solved in some models of HDE based on this fundamental assumption that matter and holographic dark energy do not conserve separately, but the matter energy density decays into the holographic energy density [77–85]. In fact a suitable evolution of the Universe is obtained when, in addition to the holographic dark energy, an interaction (decay of dark energy to matter) is assumed.

In the present paper, we suggest a correspondence between the holographic dark energy scenario and the phantom dark energy model. The current available observational data imply that the holographic vacuum energy behaves as phantom-type dark energy, i.e. the equation-of-state of dark energy crosses the cosmological-constant boundary  $w = -1$  during the evolution history. We show this phantom description of the interacting holographic dark energy in non-flat universe with  $c \leq 0.84$ , and reconstruct the potential of the phantom scalar field.

## 2 Interacting holographic phantom in non-flat universe

In this section we obtain the equation of state for the holographic energy density when there is an interaction between holographic energy density  $\rho_\Lambda$  and a cold dark matter (CDM) with  $w_m = 0$ . The continuity equations for dark energy and CDM are

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda)\rho_\Lambda = -Q, \quad (3)$$

$$\dot{\rho}_m + 3H\rho_m = Q. \quad (4)$$

The interaction is given by the quantity  $Q = \Gamma\rho_\Lambda$ . This is a decaying of the holographic energy component into CDM with the decay rate  $\Gamma$ . Taking a ratio of two energy densities as  $u = \rho_m/\rho_\Lambda$ , the above equations lead to

$$\dot{u} = 3Hu \left[ w_\Lambda + \frac{1+u}{u} \frac{\Gamma}{3H} \right]. \quad (5)$$

Following [86], if we define

$$w_\Lambda^{\text{eff}} = w_\Lambda + \frac{\Gamma}{3H}, \quad w_m^{\text{eff}} = -\frac{1}{u} \frac{\Gamma}{3H}. \quad (6)$$

Then, the continuity equations can be written in their standard form

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda^{\text{eff}})\rho_\Lambda = 0, \quad (7)$$

$$\dot{\rho}_m + 3H(1 + w_m^{\text{eff}})\rho_m = 0. \quad (8)$$

We consider the non-flat Friedmann–Robertson–Walker universe with line element

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right). \quad (9)$$

where  $k$  denotes the curvature of space  $k = 0, 1, -1$  for flat, closed and open universe respectively. A closed universe with a small positive curvature ( $\Omega_k \sim 0.01$ ) is compatible

with observations [87, 88]. We use the Friedmann equation to relate the curvature of the universe to the energy density. The first Friedmann equation is given by

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2} [\rho_\Lambda + \rho_m]. \quad (10)$$

In non-flat universe, our choice for holographic dark energy density is

$$\rho_\Lambda = 3c^2 M_p^2 L^{-2}. \quad (11)$$

$L$  is defined as the following form [65]:

$$L = ar(t), \quad (12)$$

here,  $a$ , is scale factor and  $r(t)$  is relevant to the future event horizon of the universe. Given the fact that

$$\begin{aligned} \int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}} &= \frac{1}{\sqrt{|k|}} \text{sinn}^{-1} \left( \sqrt{|k|} r_1 \right) \\ &= \begin{cases} \sin^{-1} \left( \sqrt{|k|} r_1 \right) / \sqrt{|k|}, & k = 1, \\ r_1, & k = 0, \\ \sinh^{-1} \left( \sqrt{|k|} r_1 \right) / \sqrt{|k|}, & k = -1, \end{cases} \end{aligned} \quad (13)$$

one can easily derive

$$L = \frac{a(t) \text{sinn} \left[ \sqrt{|k|} R_h(t) / a(t) \right]}{\sqrt{|k|}}, \quad (14)$$

where  $R_h$  is the future event horizon given by (2). By considering the definition of holographic energy density  $\rho_\Lambda$ , one can find [89, 90]:

$$w_\Lambda = - \left[ \frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c} \frac{1}{\sqrt{|k|}} \text{cosn} \left( \sqrt{|k|} R_h / a \right) + \frac{\Gamma}{3H} \right]. \quad (15)$$

where

$$\frac{1}{\sqrt{|k|}} \text{cosn} \left( \sqrt{|k|} x \right) = \begin{cases} \cos(x), & k = 1, \\ 1, & k = 0, \\ \cosh(x), & k = -1. \end{cases} \quad (16)$$

Here as in [95], we choose the following relation for decay rate

$$\Gamma = 3b^2(1+u)H \quad (17)$$

with the coupling constant  $b^2$ . Substitute this relation into (15), one finds the holographic energy equation of state [89]

$$w_\Lambda = -\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda - c^2\Omega_k}}{3c} - \frac{b^2(1+\Omega_k)}{\Omega_\Lambda}. \quad (18)$$

From (6), (18), we have the effective equation of state as<sup>1</sup>

$$w_\Lambda^{\text{eff}} = -\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda - c^2\Omega_k}}{3c}. \quad (20)$$

<sup>1</sup> It seems that  $w_\Lambda^{\text{eff}}$  is independent of the coupling constant  $b$ , however, a solution  $\Omega_\Lambda$  to the following evolution equation

For the non-flat universe, the authors of [96] used the data coming from the SN and CMB to constrain the holographic dark energy model, and got the  $1\sigma$  fit results:  $c = 0.84_{-0.03}^{+0.16}$ . If we take  $c = 0.84$ , and taking  $\Omega_\Lambda = 0.73$ ,  $\Omega_k = 0.01$  for the present time, using (20) we obtain  $w_\Lambda^{\text{eff}} = -1.007$ . Also for the flat case, the X-ray gas mass fraction of rich clusters, as a function of redshift, has also been used to constrain the holographic dark energy model [62]. The main results, i.e. the  $1\sigma$  fit values for  $c$  is:  $c = 0.61_{-0.21}^{+0.45}$ , in this case also we obtain  $w_\Lambda^{\text{eff}} < -1$ . This implies that one can generate phantom-like equation of state from an interacting holographic dark energy model in flat and non-flat universe only if  $c \leq 0.84$ . This upper limit on  $c$  depends on the exact value of  $\Omega_\Lambda$  and therefore depend on the coupling constant  $b$  (see (19)). Also it depend on the value of  $\Omega_k$  and cutoff  $L$ .

Now we assume that the origin of the dark energy is a phantom scalar field  $\phi$ , so

$$\rho_\Lambda = -\frac{1}{2}\dot{\phi}^2 + V(\phi) \quad (21)$$

$$P_\Lambda = -\frac{1}{2}\dot{\phi}^2 - V(\phi) \quad (22)$$

In this case  $w_\Lambda$  is given by

$$w_\Lambda = \frac{-\frac{1}{2}\dot{\phi}^2 - V(\phi)}{-\frac{1}{2}\dot{\phi}^2 + V(\phi)}. \quad (23)$$

One can see that in this case  $w_\Lambda < -1$ ,<sup>2</sup> therefore, only holographic dark energy in cases  $c \leq 0.84$  can be described by the phantom. It must be pointed out that the choice of  $c \leq 0.84$ , on theoretical level, will bring some troubles.

which includes the  $b^2$ -terms determines how the effective equation of state  $w_\Lambda^{\text{eff}}$  is changing under the evolution of the universe. The differential equation for  $\Omega_\Lambda$  is

$$\begin{aligned} \frac{d\Omega_\Lambda}{dx} &= \frac{\dot{\Omega}_\Lambda}{H} = 3\Omega_\Lambda(1+\Omega_k-\Omega_\Lambda) \\ &\times \left[ -\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda}}{3c} \frac{1}{\sqrt{|k|}} \text{cosn} \left( \sqrt{|k|} R_h / a \right) \right. \\ &\left. - b^2 \frac{(1+\Omega_k)}{\Omega_\Lambda} + \frac{b^2(1+\Omega_k)^2}{(1+\Omega_k-\Omega_\Lambda)\Omega_\Lambda} \right]. \end{aligned} \quad (19)$$

<sup>2</sup> Differentiating (10) with respect to the cosmic time  $t$ , one find

$$\dot{H} = \frac{\dot{\rho}}{6HM_p^2} + \frac{k}{a^2}, \quad (24)$$

where  $\rho = \rho_m + \rho_\Lambda$  is the total energy density, now using (3), (4)

$$\dot{\rho} = -3H(1+w)\rho \quad (25)$$

where

$$w = \frac{w_\Lambda \rho_\Lambda}{\rho} = \frac{\Omega_\Lambda w_\Lambda}{1 + \frac{k}{a^2 H^2}}. \quad (26)$$

The Gibbons–Hawking entropy will thus decrease since the event horizon shrinks, which violates the second law of thermodynamics as well. However, the current observational data indicate that the parameter  $c$  in the holographic model seems smaller than 1. Now we reconstruct the phantom potential and the dynamics of the scalar field in light of the holographic dark energy with  $c \leq 0.84$ . According to the forms of phantom energy density and pressure (21) and (22), one can easily derive the scalar potential and kinetic energy term as

$$V(\phi) = \frac{1}{2}(1 - w_\Lambda)\rho_\Lambda \quad (28)$$

$$\dot{\phi}^2 = -(1 + w_\Lambda)\rho_\Lambda. \quad (29)$$

Using (26) and (27), one can rewrite the holographic energy equation of state as

$$w_\Lambda = \frac{-1}{3\Omega_\Lambda H^2} \left( 2\dot{H} + 3H^2 + \frac{k}{a^2} \right). \quad (30)$$

Substitute the above  $w_\Lambda$  into (28) and (29), we obtain

$$V(\phi) = \frac{M_p^2}{2} \left[ 2\dot{H} + 3H^2(1 + \Omega_\Lambda) + \frac{k}{a^2} \right] \quad (31)$$

$$\dot{\phi}^2 = M_p^2 \left[ 2\dot{H} + 3H^2(1 - \Omega_\Lambda) + \frac{k}{a^2} \right]. \quad (32)$$

In the spatially flat case,  $k = 0$ , and  $\Omega_\Lambda = 1$ , in this case the (31) and (32) are exactly (6) in [91] if we consider  $\omega(\phi) = -1$ . In similar to the [91–94], we can define  $\dot{\phi}^2$  and  $V(\phi)$  in terms of single function  $f(\phi)$  as

$$V(\phi) = \frac{M_p^2}{2} \left[ 2f'(\phi) + 3f^2(\phi)(1 + \Omega_\Lambda) + \frac{k}{a^2} \right] \quad (33)$$

$$1 = M_p^2 \left[ 2f'(\phi) + 3f^2(\phi)(1 - \Omega_\Lambda) + \frac{k}{a^2} \right]. \quad (34)$$

In the spatially flat case (33) and (34) solved only in case of presence of two scalar potentials  $V(\phi)$ , and  $\omega(\phi)$ . Here we have claimed that in the presence of curvature term  $\frac{k}{a^2}$ , (33) and (34) may be solved with potential  $V(\phi)$  (To see the general procedure for such type calculations refer to [91–94], see also the appendix of the present paper). Hence, the following solution are obtained

$$\phi = t, \quad H = f(t). \quad (35)$$

Substitute  $\dot{\phi}$  into (24), we obtain

$$w = \frac{2/3 \left( \frac{k}{a^2} - \dot{H} \right)}{H^2 + \frac{k}{a^2}} - 1. \quad (27)$$

In a phantom dominated universe  $\dot{H} > 0$ , from (27) one can see easily that in the  $k = 0, k = -1$  cases  $w < -1$ , therefore in this cases  $w_\Lambda < -1$  also. For  $k = 1$ , the necessary condition to obtain  $w_\Lambda < -1$  is this:  $\dot{H} > \frac{1}{a^2}$ . However as we have shown above, with the choice of  $c \leq 0.84$ , the holographic dark energy model can predict a phantom era, even for  $k = 1$  case.

One can check that the solution (35) satisfies the following scalar field equation

$$-\ddot{\phi} - 3H\dot{\phi} + V'(\phi) = 0. \quad (36)$$

Therefore by the above condition,  $f(\phi)$  in our model must satisfy following relation

$$3f(\phi) = V'(\phi). \quad (37)$$

In the other hand, using (11) and (18) we have

$$V(\phi) = \frac{3H^2\Omega_\Lambda}{16\pi G} \left( \frac{4}{3} + \frac{2\sqrt{\Omega_\Lambda - c^2\Omega_k}}{3c} + \frac{b^2(1 + \Omega_k)}{\Omega_\Lambda} \right) \quad (38)$$

$$\dot{\phi} = \frac{H\sqrt{\Omega_\Lambda}}{2\sqrt{\pi G}} \left[ -1 + \frac{\sqrt{\Omega_\Lambda - c^2\Omega_k}}{c} + \frac{3b^2(1 + \Omega_k)}{2\Omega_\Lambda} \right]^{1/2}. \quad (39)$$

Using (39), we can rewrite (38) as

$$V(\phi) = 3M_p^2 H^2 \left( 1 + \frac{\dot{\phi}^2}{6M_p^2 H^2 \Omega_\Lambda} \right), \quad (40)$$

or in another form as following

$$V(\phi) = 3M_p^2 \left[ f^2(\phi) + \frac{1}{6M_p^2 \Omega_\Lambda} \right]. \quad (41)$$

Then, from (33) and (41), we get

$$\frac{k}{a^2} = 3f^2(\phi)(1 - \Omega_\Lambda) - 2f'(\phi) + \frac{1}{\Omega_\Lambda M_p^2}. \quad (42)$$

Now, using (34) and (42) we obtain following second order equation for  $\Omega_\Lambda$

$$6M_p^2 f^2 \Omega_\Lambda^2 + (1 - 6M_p^2 f^2) \Omega_\Lambda - 1 = 0. \quad (43)$$

The solutions of this equation are as

$$\Omega_\Lambda = \frac{(6M_p^2 f^2 - 1) \pm \sqrt{(1 - 6M_p^2 f^2)^2 + 24M_p^2 f^2}}{12M_p^2 f^2}. \quad (44)$$

Substitute the above  $\Omega_\Lambda$  into (41), we obtain the scalar potential as following

$$V(\phi) = 3M_p^2 f^2(\phi) \times \left[ 1 + \frac{2}{(6M_p^2 f^2(\phi) - 1) \pm \sqrt{(1 - 6M_p^2 f^2(\phi))^2 + 24M_p^2 f^2(\phi)}} \right]. \quad (45)$$

### 3 Conclusions

Based on cosmological state of holographic principle, proposed by Fischler and Susskind [97], the holographic model of dark energy (HDE) has been proposed and studied widely in the literature [55, 98–104]. In [105] using the type Ia supernova data, the model of HDE is constrained once when  $c$  is unity and another time when  $c$  is taken as free parameter. It is concluded that the HDE is consistent with recent observations, but future observations are needed to constrain this model more precisely. In another paper [106], the anthropic principle for HDE is discussed. It is found that, provided that the amplitude of fluctuation are variable the anthropic consideration favors the HDE over the cosmological constant. For flat universe the convenient horizon looks to be  $R_h$  while in non-flat universe we define  $L$  because of the problems that arise if we consider  $R_h$  or  $R_p$  (particle horizon) [107]. As it was mentioned in introduction,  $c$  is a positive constant in holographic model of dark energy, and ( $c \geq 1$ ). However, if  $c < 1$ , the holographic dark energy will behave like a phantom model of DE, the amazing feature of which is that the equation of state of dark energy component  $w_\Lambda$  crosses  $-1$ . Hence, we see, the determining of the value of  $c$  is a key point to the feature of the holographic dark energy and the ultimate fate of the universe as well. However, in the recent fit studies, different groups gave different values to  $c$ . A direct fit of the present available SNe Ia data with this holographic model indicates that the best fit result is  $c = 0.21$  [105]. Recently, by calculating the average equation of state of the dark energy and the angular scale of the acoustic oscillation from the BOOMERANG and WMAP data on the CMB to constrain the holographic dark energy model, the authors show that the reasonable result is  $c \sim 0.7$  [108]. In the other hand, in the study of the constraints on the dark energy from the holographic connection to the small  $l$  CMB suppression, an opposite result is derived, i.e. it implies the best fit result is  $c = 2.1$  [109]. Also, the authors of [96] used the data coming from the SN and CMB to constrain the holographic dark energy model, and got the  $1\sigma$  fit results:  $c = 0.84^{+0.16}_{-0.03}$ .

In this paper we have associated the interacting holographic dark energy in non-flat universe with a phantom scalar field. We have shown that the holographic dark energy with  $c \leq 0.84$  (here we have assumed  $c$  is a constant, to see similar calculation where  $c$  is variable with time refer to [111]) can be described by the phantom in a certain way. In the another term our calculation show, taking  $\Omega_\Lambda = 0.73$  for the present time, the upper bound of  $c$  is 0.84. Then a correspondence between the holographic dark energy and phantom has been established, and the potential of the holographic phantom has been reconstructed. This results is in contrast with some other variants of holographic dark energy discussed in the literature, for example: The authors of [86] have shown that an interacting holographic dark energy model cannot accommodate a transition from the dark energy with  $w_\Lambda^{\text{eff}} \geq -1$  to the phantom regime with  $w_\Lambda^{\text{eff}} < -1$ , also in [110] I have shown that one can not generate phantom-like equation of state from an interacting holographic dark energy model in non-flat universe in

the Brans–Dicke cosmology framework. In the other hand, in [110] I have suggested a correspondence between the holographic dark energy scenario in flat universe and the phantom dark energy model in framework of Brans–Dicke theory with potential.

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### Appendix

It has been shown in [91–94] that ark energy dynamics of the universe can be achieved by equivalent mathematical descriptions taking into account generalized fluid equations of state in general relativity, scalar-tensor theories or modified  $F(R)$  gravity in Einstein or Jordan frames.

We consider following action

$$S = \int \sqrt{-g} d^4x \left[ \frac{M_p^2}{2} R - \frac{1}{2} \omega(\varphi) \partial_\mu \varphi \partial^\mu \varphi - \tilde{V}(\varphi) \right]. \quad (\text{A.1})$$

The energy density and pressure are given by

$$\rho_\Lambda = \frac{1}{2} \omega(\varphi) \dot{\varphi}^2 + \tilde{V}(\varphi) \quad (\text{A.2})$$

$$P_\Lambda = \frac{1}{2} \omega(\varphi) \dot{\varphi}^2 - \tilde{V}(\varphi). \quad (\text{A.3})$$

According to the forms of energy density and pressure (A.2) and (A.3), one can easily derive the scalar potential and kinetic energy term as

$$\tilde{V}(\varphi) = \frac{1}{2} (1 - w_\Lambda) \rho_\Lambda \quad (\text{A.4})$$

$$\omega(\varphi) \dot{\varphi}^2 = (1 + w_\Lambda) \rho_\Lambda. \quad (\text{A.5})$$

Using (30), one can obtain

$$\tilde{V}(\varphi) = \frac{M_p^2}{2} \left[ 2\dot{H} + 3H^2(1 + \Omega_\Lambda) + \frac{k}{a^2} \right] \quad (\text{A.6})$$

$$\omega(\varphi) \dot{\varphi}^2 = -M_p^2 \left[ 2\dot{H} + 3H^2(1 - \Omega_\Lambda) + \frac{k}{a^2} \right]. \quad (\text{A.7})$$

The interesting case is that  $\omega(\varphi)$  and  $\tilde{V}(\varphi)$  are defined in terms of a single function  $f(\varphi)$  as

$$\tilde{V}(\varphi) = \frac{M_p^2}{2} \left[ 2f'(\varphi) + 3f^2(\varphi)(1 + \Omega_\Lambda) + \frac{k}{a^2} \right] \quad (\text{A.8})$$

$$\omega(\varphi) = -M_p^2 \left[ 2f'(\varphi) + 3f^2(\varphi)(1 - \Omega_\Lambda) + \frac{k}{a^2} \right]. \quad (\text{A.9})$$

Hence, the following solution are obtained

$$\varphi = t, \quad H = f(t). \quad (\text{A.10})$$

In the other hand, using (11) and (18) we have

$$\tilde{V}(\varphi) = \frac{3H^2\Omega_\Lambda}{16\pi G} \left( \frac{4}{3} + \frac{2\sqrt{\Omega_\Lambda - c^2\Omega_k}}{3c} + \frac{b^2(1+\Omega_k)}{\Omega_\Lambda} \right) \quad (\text{A.11})$$

$$\sqrt{|\omega(\varphi)|}\dot{\varphi} = \frac{H\sqrt{\Omega_\Lambda}}{2\sqrt{\pi G}} \left[ -1 + \frac{\sqrt{\Omega_\Lambda - c^2\Omega_k}}{c} + \frac{3b^2(1+\Omega_k)}{2\Omega_\Lambda} \right]^{\frac{1}{2}}. \quad (\text{A.12})$$

Using (A.12), we can rewrite (A.11) as

$$\tilde{V}(\varphi) = 3M_p^2 \left[ f^2(\varphi) - \frac{\omega(\varphi)}{6M_p^2\Omega_\Lambda} \right]. \quad (\text{A.13})$$

Then, from (A.8) and (A.13), we get

$$\frac{k}{a^2} = 3f^2(\varphi)(1-\Omega_\Lambda) - 2f'(\varphi) - \frac{\omega(\varphi)}{\Omega_\Lambda M_p^2}. \quad (\text{A.14})$$

Now, using (A.9), (A.14) we obtain following second order equation for  $\Omega_\Lambda$

$$-6M_p^2 f^2 \Omega_\Lambda^2 + (\omega(\varphi) + 6M_p^2 f^2) \Omega_\Lambda - \omega(\varphi) = 0. \quad (\text{A.15})$$

The solutions of this equation are as

$$\Omega_\Lambda = \frac{- (6M_p^2 f^2 + \omega(\varphi)) \pm \sqrt{(\omega(\varphi) + 6M_p^2 f^2)^2 - 24M_p^2 f^2 \omega(\varphi)}}{-12M_p^2 f^2}. \quad (\text{A.16})$$

Substitute the above  $\Omega_\Lambda$  into (A.13), we obtain the scalar potential as following

$$\tilde{V}(\varphi) = 3M_p^2 f^2(\varphi) \times \left[ 1 + \frac{2\omega(\varphi)}{-(6M_p^2 f^2 + \omega(\varphi)) \pm \sqrt{(\omega(\varphi) + 6M_p^2 f^2)^2 - 24M_p^2 f^2 \omega(\varphi)}} \right]. \quad (\text{A.17})$$

If we define a new field  $\phi$  as

$$\phi = \int d\varphi \sqrt{|\omega(\varphi)|} \quad (\text{A.18})$$

the action (A.1) can be rewritten as

$$S = \int \sqrt{-g} d^4x \left[ \frac{M_p^2}{2} R \mp \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]. \quad (\text{A.19})$$

The sign in front of the kinetic term depends on the sign of  $\omega(\varphi)$ . If the sign of  $\omega(\varphi)$  is positive, the sign of the kinetic term is negative. Therefore, in the phantom phase, the sign is always positive, this is what we have considered in this paper. One can assumes  $\varphi$  can be solved with respect to

$\phi$ :  $\varphi = \varphi(\phi)$ . Then the potential  $V(\phi)$  is given by  $V(\phi) = \tilde{V}(\varphi(\phi))$ . We have shown that with the choice of  $c \leq 0.84$  the interacting holographic dark energy has a phantom-like behaviour. Therefore, the equivalent scalar-tensor theory for interacting holographic fluid is given by (A.19), where the sign of the kinetic term is negative. In this paper we did not consider the potential  $\omega(\varphi)$ , we have claimed that in the presence of curvature term  $\frac{k}{a^2}$ , (33) and (34) may be solved with potential  $V(\phi)$  only, but with condition (37).

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